REAL ANALYSIS ASSIGNMENT

QUESTION 1

(a) Prove that $\sqrt{3}$ is irrational. Does a similar argument work to show $\sqrt{6}$ is irrational?

(b) Compute, without proofs, the suprema and infima of the following sets:

  i) $\{n \in N \ : \ n^2 < 10\}$

  ii) $\{\frac{n}{n+m} \ : \ n, m \in N\}$

  iii) $\{\frac{n}{2n+1} \ : \ n \in N\}$

  iv) $\{\frac{n}{m} \ : \ n, m \in N$ with $m + n \leq 10\}$

(c) Without worrying about formal proofs for the moment, decide if the following statements about suprema and infima are true or false. For any that are false, supply an example where the claim in question does not appear to hold.

  i) A finite, nonempty set always contains its supremum.

  ii) If $a < L$ for every element a in the set A, then $\sup A < L$.

  iii) If A and B are sets with the property that $a < b$ for every $a \in A$ and every $b \in B$, then it follows that $\sup A < \inf B$.

  iv) If $\sup A = s$ and $\sup B = t$, then $\sup(A + B) = s + t$. The set $A + B$ is defined as $A + B = \{a + b : a \in A$ and $b \in B\}$.

  v) If $\sup A \leq \sup B$, then there exists an element $b \in B$ that is an upper bound for A.

(d) Verify, using the definition of convergence of a sequence, thatthe following sequences converge to the proposed limit.

  (i) $\lim\limits_{n\to\infty} \dfrac{1}{6n^2 + 1} = 0.$

  (ii) $\lim\limits_{n\to\infty} \dfrac{3n + 1}{2n + 5} = \dfrac{3}{2}.$

  (iii) $\lim\limits_{n\to\infty} \dfrac{2}{\sqrt{n + 3}} = 0.$

(e) Let $\lim\limits_{n\to\infty} a_n = a$, and $\lim\limits_{n\to\infty} b_n = b$. Prove that

  (i) $\lim\limits_{n\to\infty} ca_n = ca$ for all $c \in \mathbb{R}$.

  (ii) $\lim\limits_{n\to\infty} (a_n + b_n) = a + b$.

  (iii) $\lim\limits_{n\to\infty} (a_n b_n) = ab$.

(e) Show that if $b_n \to b$, then the sequence of absolute values $|b_n|$ converges to $|b|$.

(f) Is the converse of part (e) true? If we know that $|b_n| \to |b|$, can we deduce that $b_n \to b$?

(g) If $a_n \to 0$ and $|b_n - b| \leq a_n$, then show that $b_n \to b$.